

## LORENTZ VIOLATION AS A QUANTUM-GRAVITY SIGNATURE\*

RALF LEHNERT

*CENTRA, Departamento de Física, Universidade do Algarve  
8000-117 Faro, Portugal  
E-mail: rlehnert@ualg.pt*

Many theoretical approaches to quantum gravity predict the breakdown of Lorentz symmetry at Planck energies. Kinematical cosmic-ray studies are a sensitive tool in the search for such effects. This talk discusses the construction of test dispersion relations for such analyses.

### 1. Introduction

One of the most fundamental problems in present-day physics concerns a quantum theory of gravitation. Such a theory is believed to be associated with the Planck scale  $M_{Pl}$ , so that quantum-gravity signatures are likely to be suppressed by one or more powers of  $M_{Pl}$  at currently attainable energies. The expected minuscule size of potential effects and the absence of a fully realistic and satisfactory underlying theory make quantum-gravity phenomenology particularly challenging. A practical approach to overcome this obstacle is to consider the breaking of symmetries that hold exactly in our current fundamental laws, might be violated in approaches to quantum gravity, and can be tested with ultrahigh precision.

Lorentz- and CPT-symmetry breakdown offers a promising tool in this line of investigation.<sup>1</sup> An effective-field-theory framework for the description of such effects at present experimental energy scales is provided by the Standard-Model Extension (SME).<sup>2,3,4</sup> The SME coefficients parametrizing Lorentz and CPT violation can arise in numerous approaches to quantum gravity including strings,<sup>5</sup> spacetime-foam models,<sup>6,7,8</sup> noncommu-

---

\*Funded in part by the Centro Multidisciplinar de Astrofísica (CENTRA) and by the Fundação para a Ciência e a Tecnologia (Portugal) under grant POCTI/FNU/49529/2002.

tative field theory,<sup>9</sup> cosmologically varying scalars,<sup>10,11</sup> random-dynamics models,<sup>12</sup> multiverses,<sup>13</sup> and brane-world scenarios.<sup>14</sup>

The flat-spacetime limit of the SME has provided the basis for numerous experimental and theoretical investigations of Lorentz and CPT breakdown, many of which place tight constraints on SME parameters. Examples of such analyses include ones involving mesons,<sup>15,16,17,18</sup> baryons,<sup>19,20,21</sup> electrons,<sup>22,23,24</sup> photons,<sup>10,25,26,27,28</sup> and muons.<sup>29</sup> We also remark that neutrino-oscillation experiments offer the potential for discovery.<sup>2,30,31</sup>

The quadratic sector of the SME determines the one-particle dispersion relations, which typically exhibit Lorentz-violating modifications,<sup>25,2,4</sup> as expected. It follows that particle-reaction kinematics is altered relative to the conventional case leading to potentially observable shifts in certain reaction thresholds. At energies approaching the Planck scale, such Lorentz-violating effects might be more pronounced, so that kinematical investigations involving ultrahigh-energy cosmic rays (UHECR) can provide a high-sensitivity laboratory for Lorentz-violation searches.

Many recent quantum-gravity investigations outside the context of the SME have exploited the general idea to constrain deviations from conventional dispersion relations.<sup>32,33</sup> Because of the absence of a complete underlying theory, the form of the considered Lorentz-breaking dispersion-relation modifications appears somewhat arbitrary in some of these studies. This talk addresses the question as to whether at least fundamental physics principles that are expected to remain valid in an underlying theory can yield meaningful restrictions on possible dispersion-relation modifications. We primarily consider the principle of coordinate independence and the requirement of compatibility with an effective dynamical framework. We explain why these two conditions should continue to hold in the presence of Lorentz violation regardless of the details of the Planck-scale theory and discuss the ensuing constraints on dispersion relations. Throughout we assume translational invariance and the associated energy-momentum conservation.

The outline of this talk is as follows. Section 2 discusses the need for coordinate independence and its consequences. In Sec. 3, we comment on the compatibility of the modified dispersion relations with dynamics. Some subtle issues regarding the applicability of certain conventional approximations in the Lorentz-violating context are addressed in Sec. 4.

## 2. Coordinate independence

Although coordinate systems are one of the most important mathematical tools in physics, they do not possess physical reality: the description of a process must remain independent of the choice of coordinates. This allows, for example, different observers, each equipped with a different reference frame, to relate their observations or predictions concerning a given physical system. The principle of coordinate independence is therefore also called observer invariance. Mathematically, observer invariance is usually implemented by choosing a spacetime-manifold description for physical events, tensors or spinors for the representation of observables, and covariant equations as the laws of physics.

It is a common misconception that Lorentz breaking implies the loss of coordinate independence. Such a model would inhibit meaningful physical predictions, so that it is necessary to adhere to observer invariance also in the presence of Lorentz violation. This is by no means a contradiction. Consider, for instance, the conventional motion of a classical point particle of mass  $m$  and charge  $q$  in an external electromagnetic field  $F^{\mu\nu}$  described by

$$m \frac{dv^\mu}{d\tau} = q F^{\mu\nu} v_\nu . \quad (1)$$

Here,  $\tau$  denotes the proper time and  $v^\mu$  the four-velocity of the particle. Note that invariance under rotations of the particle's trajectory is broken by the external nondynamical  $F^{\mu\nu}$  resulting in the nonconservation of the charge's angular momentum. However, Eq. (1) is a tensor equation valid in all coordinate systems maintaining coordinate independence. This example also illustrates that transformations of localized dynamical particles and fields (with nondynamical global backgrounds held fixed) must be clearly distinguished from transformations of the coordinates. The former, also called particle transformations, are no longer associated with a symmetry of the model.

In what follows, we consider models that exhibit particle Lorentz violation while maintaining coordinate independence. In many respects, the physical situation is conceptually similar to the external-field case discussed in the previous paragraph. However, in the above example the  $F^{\mu\nu}$  background is a local electromagnetic field generated by other charge and current distributions that can in principle be controlled. In the present Lorentz-violating context, such a background is a global property of the effective vacuum outside of experimental control.

We continue our study in a local Minkowski frame. Lorentz-violating dispersion relations are usually taken to be of the form

$$E_0^2 - \vec{p}^2 = m^2 + \delta f(E, \vec{p}) , \quad (2)$$

where  $p^\mu = (E, \vec{p})$  and  $m$  are the particle's respective four-momentum and mass, and  $\delta f(E, \vec{p})$  describes the Lorentz violation. With our above consideration, the correction  $\delta f$  needs to be observer Lorentz invariant, i.e., it must transform as a scalar under coordinate changes. To obtain a general form of  $\delta f$ , we impose some further mild conditions. First, for small momenta<sup>a</sup>  $|\vec{p}| \ll M_{Pl}$  we want to recover the usual dispersion relation with  $\delta f = 0$ . Second, we want to avoid potential nonlocalities that could arise through the presence of nonpolynomial functions. This yields the ansatz

$$\delta f(E, \vec{p}) = \sum_{n \geq 1} \overbrace{T_{(n)}^{\alpha\beta \cdots}}^{n \text{ indices}} \underbrace{p_\alpha p_\beta \cdots}_{n \text{ factors}} . \quad (3)$$

Here,  $T_{(n)}^{\alpha\beta \cdots}$  is a constant tensor of rank  $n$  describing particle Lorentz violation. All the tensor indices  $\alpha, \beta \dots$  are distinct, and each one is properly contracted with a momentum factor, so that each term in the sum is coordinate independent.

This ansatz has various consequences,<sup>33</sup> two of which we mention next: when viewed as a polynomial in  $E$ , the modified dispersion relation (2) will in general lift the usual degeneracy between particle, antiparticle, and possible spin-type states. Another implication concerns cases with imposed rotational invariance. Then, the general ansatz (3) does not contain odd powers of  $|\vec{p}|$ .<sup>33</sup>

### 3. Effective Field Theory

Kinematical considerations can impose powerful restrictions on particle reactions. However, dispersion-relation constraints can be masked by other effects. For example, a high-energy reaction expected to be suppressed by a modified dispersion relation, could also be prevented by a novel symmetry. Similarly, the presence at high energies of a reaction kinematically forbidden at low energies might perhaps be explained by additional channels due

<sup>a</sup>Here,  $\vec{p}$  refers to the components in any frame in which the Earth moves nonrelativistically.

to new undetected particles or the loss of low-energy symmetries. In addition, models of both acceleration mechanisms for UHECRs and atmospheric shower development involve conventional dynamics.

We see that the implementation of general dynamical features both appears necessary for a complete description of UHECR physics and can increase the scope of cosmic-ray analyses. At the same time, it may introduce a certain degree of framework dependence. However, implementing dynamics is tightly constrained by the requirement that known physics must be recovered in certain limits. In what follows, we argue that effective field theory (EFT) is a sensible and general approach for such efforts.

EFTs have proved to be extremely successful in describing diverse physical systems at atomic, nuclear, and elementary-particle scales. Note in particular, that EFT is flexible enough for applications involving discrete condensed-matter backgrounds, situations analogous to those in some quantum-gravity approaches. Moreover, the usual Standard Model itself is normally viewed as an EFT approximating more fundamental physics.

The construction of a suitable EFT can employ a philosophy paralleling that of the dispersion-relation approach in the previous section with the more powerful idea of proceeding at the Lagrangian level: one adds Lorentz-breaking terms  $\delta\mathcal{L}$  to the usual Standard-Model Lagrangian  $\mathcal{L}_{\text{SM}}$

$$\mathcal{L}_{\text{SME}} = \mathcal{L}_{\text{SM}} + \delta\mathcal{L} , \quad (4)$$

where  $\mathcal{L}_{\text{SME}}$  denotes the EFT Lagrangian. The correction  $\delta\mathcal{L}$  is formed by contracting Standard-Model field operators of unrestricted dimensionality with Lorentz-breaking tensorial parameters (analogous to the  $T_{(n)}^{\alpha\beta\cdots}$  in Eq. (3)) yielding observer Lorentz scalars. This general EFT for Lorentz violation is the SME mentioned in the introduction. Its apparent generality makes it difficult and perhaps even impossible to find some other effective theory for Lorentz breaking containing the Standard Model with dynamics significantly different from the SME.

#### 4. Additional considerations

In this section, we point out that care is required when approximating Lorentz-violating dispersion relations. This is best demonstrated with a specific example.

Consider the modified dispersion relation

$$E^2 - \vec{p}^2 = m^2 + \frac{E\vec{p}^2}{M} , \quad (5)$$

where  $M$  is some high-energy scale, such as  $M_{Pl}$ . The usual quantum-field reinterpretation of the negative-energy solutions yields the respective exact particle and antiparticle energies  $E_+$  and  $E_-$ :

$$E_{\pm}(\vec{p}) = \sqrt{\frac{\vec{p}^4}{4M^2} + \vec{p}^2 + m^2} \pm \frac{\vec{p}^2}{2M}. \quad (6)$$

If one instead employs the ultrarelativistic approximation  $E \simeq |\vec{p}|$  in the correction term, as is sometimes done in such kinematical studies, one obtains  $\delta f \simeq |\vec{p}|^3/M$ . After the reinterpretation, we now have

$$E_{\pm}(\vec{p}) = \sqrt{\frac{|\vec{p}|^3}{M} + \vec{p}^2 + m^2}. \quad (7)$$

Next, we look at photon decay into an electron–positron pair<sup>b</sup>  $\gamma \rightarrow e^+ + e^-$  assuming a dispersion relation of the type (5) for both the leptons and the photon. For each particle, we take  $M = -M_{Pl}$ , and for the photon we set  $m = 0$ . Using the exact expression for the particle energies (6), one has to consider two distinct incoming photon states  $\gamma_+$  and  $\gamma_-$ , where the subscripts correspond to those of the particle energy in Eq. (6). One can then show<sup>33</sup> that the decay  $\gamma_- \rightarrow e^+ + e^-$  is allowed above a certain threshold. If the observed value  $m = 0.511$  MeV for the electron and positron masses is used, the numerically determined threshold value for the incoming photon three-momentum is  $|\vec{p}_{\min}| \simeq 7.21$  TeV. If one instead employs the approximate particle energies (7), photon decay is forbidden throughout the validity range of Eq. (7).

Considering the TeV threshold scale for this decay and the Planck suppression of the correction  $\delta f$ , the ultrarelativistic approximation is indeed excellent if one is interested in the particle energy only. However, threshold analyses are based on exact energy–momentum conservation and can thus be sensitive to the slightest deviations. Even in a conventional photon decay, the ultrarelativistic approximation renders the lepton momenta lightlike, which seemingly permits the decay in forward direction. In the present case,  $E \simeq |\vec{p}|$  introduces an additional degeneracy into the problem. As a result, the approximate solution is spacelike, whereas the exact expression (6) determines both a timelike and a spacelike branch. It is the presence of the timelike momenta that permits the decay.

<sup>b</sup>This process is kinematically forbidden in conventional physics.

## References

1. For an overview see, e.g., *CPT and Lorentz Symmetry II*, edited by V.A. Kostelecký (World Scientific, Singapore, 2002).
2. D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); **58**, 116002 (1998).
3. V.A. Kostelecký, Phys. Rev. D **69**, 105009 (2004); R. Bluhm and V.A. Kostelecký, Phys. Rev. D **71**, 065008 (2005).
4. V.A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001); R. Lehnert, J. Math. Phys. **45**, 3399 (2004); D. Colladay and P. McDonald, Phys. Rev. D **70**, 125007 (2004); B. Altschul and D. Colladay, Phys. Rev. D **71**, 125015 (2005).
5. V.A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); Phys. Rev. Lett. **66**, 1811 (1991); V.A. Kostelecký and R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Lett. B **381**, 89 (1996); Phys. Rev. D **63**, 046007 (2001); V.A. Kostelecký, M. Perry, and R. Potting, Phys. Rev. Lett. **84**, 4541 (2000).
6. R. Gambini and J. Pullin, in Ref. 1; J. Alfaro *et al.*, Phys. Rev. D **66**, 124006 (2002); D. Sudarsky *et al.*, Phys. Rev. Lett. **89**, 231301 (2002); Phys. Rev. D **68**, 024010 (2003); G. Amelino-Camelia, Mod. Phys. Lett. **A18**, 1073 (2003); R. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003); N.E. Mavromatos, Nucl. Instrum. Meth. B **214**, 1 (2004).
7. See, however, C.N. Kozameh and M.F. Parisi, Class. Quant. Grav. **21**, 2617 (2004).
8. C. Adam and F.R. Klinkhamer, Nucl. Phys. B **607**, 247 (2001); F.R. Klinkhamer and C. Rupp, Phys. Rev. D **70**, 045020 (2004); F.R. Klinkhamer and G.E. Volovik, Int. J. Mod. Phys. A **20**, 2795 (2005).
9. S.M. Carroll *et al.*, Phys. Rev. Lett. **87**, 141601 (2001); Z. Guralnik *et al.*, Phys. Lett. B **517**, 450 (2001); C.E. Carlson, C.D. Carone, and R.F. Lebed, Phys. Lett. B **518**, 201 (2001); A. Anisimov *et al.*, Phys. Rev. D **65**, 085032 (2002); I. Mocioiu, M. Pospelov, and R. Roiban, Phys. Rev. D **65**, 107702 (2002); J.L. Hewett, F.J. Petriello, and T.G. Rizzo, Phys. Rev. D **66**, 036001 (2002).
10. V.A. Kostelecký, R. Lehnert, and M.J. Perry, Phys. Rev. D **68**, 123511 (2003); O. Bertolami, R. Lehnert, R. Potting and A. Ribeiro, Phys. Rev. D **69**, 083513 (2004).
11. N. Arkani-Hamed *et al.*, JHEP **0405**, 074 (2004).
12. C.D. Froggatt and H.B. Nielsen, hep-ph/0211106.
13. J.D. Bjorken, Phys. Rev. D **67**, 043508 (2003).
14. C.P. Burgess *et al.*, JHEP **0203**, 043 (2002); A.R. Frey, JHEP **0304**, 012 (2003); J. Cline and L. Valcárcel, JHEP **0403**, 032 (2004).
15. KTeV Collaboration, H. Nguyen, in Ref. 1; OPAL Collaboration, R. Akkerstaff *et al.*, Z. Phys. C **76**, 401 (1997); DELPHI Collaboration, M. Feindt *et al.*, preprint DELPHI 97-98 CONF 80 (1997); BELLE Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **86**, 3228 (2001); BaBar Collaboration, B. Aubert *et al.*, hep-ex/0303043; FOCUS Collaboration, J.M. Link *et al.*, Phys. Lett. B **556**, 7 (2003).

16. V.A. Kostelecký and R. Potting, Phys. Rev. D **51**, 3923 (1995).
17. D. Colladay and V.A. Kostelecký, Phys. Lett. B **344**, 259 (1995); Phys. Rev. D **52**, 6224 (1995); Phys. Lett. B **511**, 209 (2001); V.A. Kostelecký and R. Van Kooten, Phys. Rev. D **54**, 5585 (1996); O. Bertolami *et al.*, Phys. Lett. B **395**, 178 (1997); N. Isgur *et al.*, Phys. Lett. B **515**, 333 (2001).
18. V.A. Kostelecký, Phys. Rev. Lett. **80**, 1818 (1998); Phys. Rev. D **61**, 016002 (2000); Phys. Rev. D **64**, 076001 (2001).
19. L.R. Hunter *et al.*, in *CPT and Lorentz Symmetry*, edited by V.A. Kostelecký (World Scientific, Singapore, 1999). D. Bear *et al.*, Phys. Rev. Lett. **85**, 5038 (2000); D.F. Phillips *et al.*, Phys. Rev. D **63**, 111101 (2001); M.A. Humphrey *et al.*, Phys. Rev. A **68**, 063807 (2003); Phys. Rev. A **62**, 063405 (2000); V.A. Kostelecký and C.D. Lane, Phys. Rev. D **60**, 116010 (1999); J. Math. Phys. **40**, 6245 (1999); C.D. Lane, Phys. Rev. D **72**, 016005 (2005).
20. R. Bluhm *et al.*, Phys. Rev. Lett. **88**, 090801 (2002); Phys. Rev. D **68**, 125008 (2003).
21. F. Canè *et al.*, Phys. Rev. Lett. **93**, 230801 (2004).
22. H. Dehmelt *et al.*, Phys. Rev. Lett. **83**, 4694 (1999); R. Mittleman *et al.*, Phys. Rev. Lett. **83**, 2116 (1999); G. Gabrielse *et al.*, Phys. Rev. Lett. **82**, 3198 (1999); R. Bluhm *et al.*, Phys. Rev. Lett. **82**, 2254 (1999); Phys. Rev. Lett. **79**, 1432 (1997); Phys. Rev. D **57**, 3932 (1998).
23. B. Heckel, in Ref. 1; L.-S. Hou, W.-T. Ni, and Y.-C.M. Li, Phys. Rev. Lett. **90**, 201101 (2003); R. Bluhm and V.A. Kostelecký, Phys. Rev. Lett. **84**, 1381 (2000).
24. H. Müller *et al.*, Phys. Rev. D **68**, 116006 (2003).
25. S.M. Carroll, G.B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990); M.P. Haugan and T.F. Kauffmann, Phys. Rev. D **52**, 3168 (1995); V.A. Kostelecký and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).
26. R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999); H. Müller *et al.*, Phys. Rev. D **67**, 056006 (2003); V.A. Kostelecký and A.G.M. Pickering, Phys. Rev. Lett. **91**, 031801 (2003); G.M. Shore, Contemp. Phys. **44**, 503 (2003); B. Altschul, Phys. Rev. D **69**, 125009 (2004); D. Colladay and P. McDonald, J. Math. Phys. **45**, 3228 (2004); R. Lehnert and R. Potting, Phys. Rev. Lett. **93**, 110402 (2004); Phys. Rev. D **70**, 125010 (2004) [Erratum-ibid. D **70**, 129906 (2004)].
27. J. Lipa *et al.*, Phys. Rev. Lett. **90**, 060403 (2003); H. Müller *et al.*, Phys. Rev. Lett. **91**, 020401 (2003); P. Wolf *et al.*, Gen. Rel. Grav. **36**, 2352 (2004).
28. V.A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).
29. V.W. Hughes *et al.*, Phys. Rev. Lett. **87**, 111804 (2001); R. Bluhm *et al.*, Phys. Rev. Lett. **84**, 1098 (2000).
30. S. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1999); V. Barger *et al.*, Phys. Rev. Lett. **85**, 5055 (2000); J.N. Bahcall, V. Barger, and D. Marfatia, Phys. Lett. B **534**, 114 (2002); V.A. Kostelecký and M. Mewes, Phys. Rev. D **70**, 031902 (2004); S. Choubey and S.F. King, Phys. Lett. B **586**, 353 (2004).
31. V.A. Kostelecký and M. Mewes, Phys. Rev. D **69**, 016005 (2004).
32. See, e.g., W. Kluźniak, Astropart. Phys. **11**, 117 (1999); astro-ph/9905308;



- T. Kifune, *Astrophys. J.* **518**, L21 (1999); O. Bertolami and C.S. Carvalho, *Phys. Rev. D* **61**, 103002 (2000); T. Jacobson, S. Liberati, and D. Mattingly, *Phys. Rev. D* **66**, 081302 (2002); T.J. Konopka and S.A. Major, *New J. Phys.* **4**, 57 (2002); H. Vankov and T. Stanev, *Phys. Lett. B* **538**, 251 (2002); M. Jankiewicz *et al.*, *Astropart. Phys.* **21**, 651 (2004); O. Gagnon and G.D. Moore, *Phys. Rev. D* **70**, 065002 (2004).
33. R. Lehnert, *Phys. Rev. D* **68**, 085003 (2003); hep-ph/0402133.